

# Redundancy: A Method for System Reliability Evaluation

## Abstract

Reliability is a ability of equipment to preserve its output characteristic parameters within established limits under given operating condition. Several researchers and mathematicians have been looking for various systems those can improve the efficiency of the instruments used in various industries. Reliability of any instrument is directly related to the manufacturing cost and to serve this purpose , redundancy has come up as a efficient tool which can increase the reliability. Redundancy is very effective method to improve system reliability over the other methods.

**Keywords:** Reliability, Redundancy, Efficiency, Flow Chart.

## Introduction

These days we need highly sophisticated electronic goods, radars, airborne system, rockets etc but we need reliable equipments, which requires the necessity to design reliable system.

Complexity and automation of equipment used in the industry resulted in severe problems of maintenance and repair. Failures of equipment and component increases the manufacturing cost. Failure of the unit means the unit is not providing intended function. The cause factor may be environment, conditions, temperature, humidity, vibrations etc. The reliability can be increased by using one or more parallel similar components.

Reliability means – probability that a system will not be failure during a specified time. If it is used under given specified condition reliability requires four components -

- (a) Probability
- (b) Intended function
- (c) Time
- (d) Operating Conditions

If  $T$  is the time up to which no failure occurs then

$$R(t) = P(T > t) \text{ (Hence reliability is always a function of}$$

$$\text{time) } R(0) = 1, \quad R(\infty) = 0.$$

In recent past, several researches have evaluated reliability of several complex systems consisting two or more classes by introducing redundancy as well as repair facilities.

## Aim of the Study

The primary aim of the study was to evaluate the possibilities of projecting redundancy as an effective tool to increase the reliability of instruments.

## Tools

Redundancy is very effective method to improve system reliability over the other methods. This can provide desired level of reliability.

1. Any desired level of reliability can be achieved (if the available sources permit)
2. Increases reliability of per unit resource is highest when optimal redundancy techniques are employed.
3. Design via redundancy may be obtained even if the designer is of less skill
4. It provides a quick solution.
5. This method can be employed in case of failure of all other methods.

There are various redundancy employed such as depending upon feasibility.

1. Active (Hot) redundancy
2. Stand by redundancy
3. Warm redundancy
4. Component redundancy



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5. Hierarchical redundancy etc.

A redundancy system having  $m$  units in parallel are referred to as  $m$ -order system. The performance requirements of such system may impose that at-least minimum of  $K$  units out of  $m$  units should be in operational condition for success of system.

These  $k$  are known as basic units and  $m - k$  are known as redundant units which are added to increase system reliability. Such systems are classified as  $k$  - out of  $m$  systems. There may be equipment hierarchically in the system which is often found in system containing many stages in series such as:

1. Communication
2. Data processing
3. Electrical power distribution

**Result and findings**

In a series parallel at least one element in each stage must be operable for the successful operation of the system. If  $m_i$  element each with unreliability  $q_i$  are used at stage  $i$ , then the reliability of the entire system having  $n$  stage is

$$(i) \quad R = \prod_{i=1}^n R_i(m_i) \\ = \prod_{i=1}^n [1 - Q_i(m_i)] \\ = \prod_{i=1}^n [1 - q_i^{m_i}]$$

and the unreliability is

$$(ii) \quad Q = 1 - \prod_{i=1}^n [1 - q_i^{m_i}]$$

When  $q_i^{m_i}$  are small, their products may be neglected. Then,

$$(iii) \quad R = 1 - \sum_{i=1}^n q_i^{m_i}$$

and

$$(iv) \quad Q = \prod_{i=1}^n q_i^{m_i}$$

The total reliability in a multi constraint problem is limited by following constraints.

$$(v) \quad \sum_{i=1}^n C_{ij} m_i \leq C_j, \quad j = 1, 2, \dots, k$$

where  $C_{ij}$  is  $j^{\text{th}}$  resource used at the  $i^{\text{th}}$  stage. The constraints might represent total cost ( $C_1$ ), total volume ( $C_2$ ), etc. Equation (iii) and (iv) are objective function and Eq. (v) is constraint function. Now the problem can be stated as follows. Find the vector  $M = (m_1, m_2, \dots, m_n)$  which shall:

$$(vi) \text{ (a) Maximize } Q = \prod_{i=1}^n (1 - q_i^{m_i})$$

$$(b) \text{ Subject to } m_i \geq 1, \quad i = 1, 2, \dots, n$$

$$C_{ij} > 0, \quad j = 1, 2, \dots, k$$

$$\sum C_{ij} m_i \leq c_j$$

Since  $m_i$  is assumed to be a continuous variable, the maximum  $R$  can be found by differentiating the eq. (i) with respect to  $m_i$  and equating to zero. From Eq.(i) we have

$$(vii) \quad \log R = \sum_{i=1}^n \log(1 - q_i^{m_i})$$

$$(viii) \quad \frac{1}{R} \frac{dR}{dm_i} = - \frac{q_i^{m_i} \log(q_i)}{(1 - q_i^{m_i})}$$

$$(ix) \quad \frac{dR}{dm_i} = \frac{q_i^{m_i} \log(q_i)}{(1 - q_i^{m_i})} \prod_{i=1}^n (1 - q_i^{m_i}) \\ q_i^{m_i} \log(q_i) \prod_{j=1}^n (1 - q_j^{m_j})$$

Neglecting the products of  $q^m$ , Eq. (ix) can be written as

$$\frac{dR}{dm_i} = - q_i^{m_i} \log(q_i), \quad i = 1, 2, \dots, n$$

The optimal value of  $m_i = m_i^*$  is the solution of

$$\frac{dR}{dm_i} = - q_i^{m_i} \log(q_i), \quad i = 1, 2, \dots, n$$

i.e. (x)

$$q_1^{m_1} \log(q_1) = q_2^{m_2} \log(q_2) = q_n^{m_n} \log(q_n)$$

Assuming that  $m_1^*$  is known, other values

$m_2^*, m_3^*, \dots, m_n^*$  can be determined from

$$(xi) \quad m_i = \frac{\log \frac{(E \times H)}{G}}{G}, \quad i = 1, 3, \dots, n$$

where,

- (a)  $E = q_1^{m_1^*}$
- (b)  $G = \log q_i, \quad i = 2, 3, \dots, n$
- (c)  $H = \log q_1$

Eq. (xii) can be written as

(xiii)

$$f_j = \sum_{i=1}^n (c_{ij} m_i - c_j) \leq 0, \quad j = 1, 2, \dots, k .$$

**Findings**

Following flow chart may be help in finding out the optimal redundancy of the instruments used in any system.

The detailed procedure for computing the vector  $M$  is given below :

**Step 1:** Choose the initial value of  $m_{1+}$

**Step 2:** Find  $m_i^*$ ,  $i = 2, \dots, n$

**Step 3:** Check cost and volume.

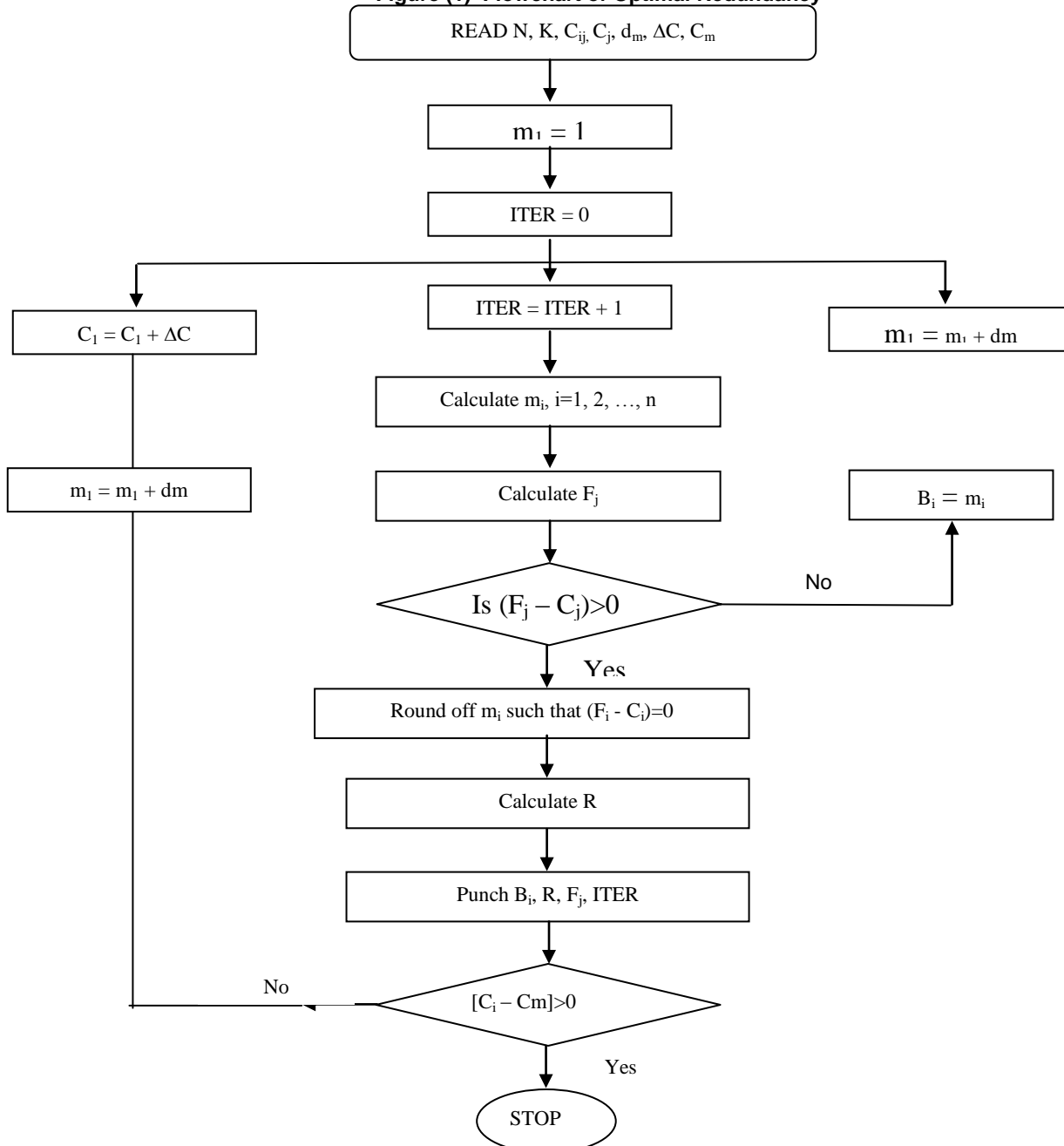
If they are not violated, go to step 4, otherwise go to step 5.

**Step 4:** Give a small positive increment  $dm$  to  $m_i^*$  go to step 5.

**Step 5:** Round off  $m_i^*$  such that the constraints are not violated.

**Step 6:** Calculated system reliability, cost and volume. A flow chart is shown in Fig. (1)

**Figure (1) Flowchart of Optimal Redundancy**



**Discussion and Suggestion**

Singh and Billinton(1974) used this technique to solve this method in steel industry. They also introduced concept of reduced efficiency state of system/equipment. Since then Govil (1971), Gupta (1973), Gopalan (1973), Linton and Saw (1974), Kantoleon (1974), Khalil (1973), Qsaki and Nakagama (1975-76), Sharma (1978), Gupta (1978), Morison and Milton (1981), Dhillon (1981), Agarwal (1984), Sharma Veeresh (2000), Shailza (2002), Shrikant (2004), Balagurusamy (2003), Mishra (1973), Ebling (2005) etc. has a very creative role and also produced a lot of literature.

**Conclusion**

Redundancy is very effective method to improve system reliability over the other methods. This can provide desired level of reliability. This method may result in increases reliability of per unit resource is highest when optimal redundancy techniques are employed. Moreover it may help industries finding better design via redundancy may be obtained even if the designer is of less skill.

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